

Holographic Implementation of the Optical Theorem

Edwin A. Marengo, Marcus V. Bunn, and Jing Tu

Abstract—This work provides an overview of recent advances in the theory and imaging applications of the optical theorem. The latter is a fundamental result of electromagnetics, acoustics, and quantum-mechanical scattering, with many important applications in imaging and detection. Particular emphasis is given to our latest efforts aimed at developing the holographic implementation of the optical theorem for the measurement of the power extincted by a scatterer that is interrogated by an arbitrary probing field and is embedded in an arbitrary medium or scene. Several practical configurations of optical theorem detectors are developed in the paper. This includes a new lens-imaging-based optical theorem detector and different forms of optical theorem holograms with particular emphasis on Leith-Upatnieks-holography-inspired optical theorem detectors. The developed optical theorem holography is particularly relevant for optical and quantum systems in which one measures only field intensities, and has many potential applications including sensors of constitutive properties of materials, optical scanners, biometric readers, communication systems, and information storage devices. The optical theorem holography theory developed in the paper is illustrated and validated with a canonical scattering example in two-dimensional space.

Index Terms—scattering theory, scattering measurements, inverse scattering, holography, optical theorem, optical detection.

I. INTRODUCTION

The optical theorem is a basic result of wave scattering theory, which applies to electromagnetic, acoustic, and quantum-mechanical scattering. In its most familiar form, which is applicable for plane wave excitation and free space, it relates the total extinction cross section of the scatterer due to both dissipation and scattering (into all directions) to the imaginary part of the forward scattering amplitude [1], [2], [3]. This fundamental result was first obtained by Wolfgang Von Sellmeier and Lord Rayleigh (John Strutt) in 1871. Lord Rayleigh used it in his well known investigations on the color and polarization of the sky. The theorem was later extended to quantum scattering theory by other researchers such as Wheeler [4] and Bohr *et al.* [5], and was known for some time as “the Bohr-Peierls-Placzek relation”. In 1952, this theorem was first called “a well-known theorem of optics” in a paper published by Rohrllich and Gluckstern [6]. It was first referred to as “the optical theorem” in a book published by Bethe, De Hoffman, and Schweber in 1955 [7]. The optical theorem has played a major role in the theories of electromagnetics, optics, acoustics, and quantum mechanics (see [8], [9], [10], [11], [12], [13], [14] for representative treatments), and has

been at the heart of many important applications in sensing and imaging (see, e.g., [15], [16], [17]), and in the testing of computational codes ([18], [19]), among other areas.

In recent years, both the theory and practical applications of the optical theorem have expanded significantly. Recent theoretical developments in this area include the unified Green’s operator theory in [20], fundamental connections to Green’s function extraction from field correlations [21], [22], generalizations to arbitrary probing fields and media [16], [23], [24], including extensions to nonhomogeneous media [25], surface waves and layered media [26], anisotropic and nonreciprocal media [27], open waveguides [28], transmission lines [29], time-varying and nonlinear scatterers [30], [31], active scatterers [32] and so on, as well as formulations for specialized sensing geometries and coordinate systems [33], [34], [35]. In recent years, the optical theorem has also found further practical applications in many fields, including lasers [36], metamaterials [37], sensors [38], [39], computational methods [40], and change detection [32], [41].

One of the most interesting areas that has emerged in recent years is the application of the optical theorem for detection [32], [41] and the associated generalization of the optical theorem to arbitrary fields and media [23], [24], [27], [32]. Some of these investigations [27], [32], [41] have led to a quite general recipe for the design of optical theorem sensors that measure the extincted power for arbitrary fields and media. This particular approach relies on the use of time reversal mirrors or cavities which is suitable for acoustics and electromagnetics. On the other hand, this method relies on the sensing of signal phase information, and is therefore not suitable for optical and quantum systems. In particular, in the optical regime one measures only field intensities. As a consequence, the corresponding optical implementation, be it analogically or digitally, of the required time reversal mirror, is less obvious.

In this work we provide an overview of our latest efforts aimed at addressing this challenge. We discuss the use of optical holography to develop optical theorem sensors that measure the power extincted by a scatterer that is interrogated by an arbitrary probing field and is embedded in an arbitrary medium or scene that can be unknown and quite complex. For simplicity, we present our developments in the basic framework of the scalar Helmholtz equation. The scatterer is modeled as a nonhomogeneous and possibly lossy perturbation of the background which is assumed to be nonhomogeneous but lossless. The probing field used to interrogate the scatterer is general and can include, e.g., near field, evanescent components such as in near field imaging applications. Prior related work is that of Berg *et al.* [42] which illustrates an approach to measure power extinction using holography. However, the method in [42] holds only

E. A. Marengo is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, Massachusetts 02115, USA, email: emarengo@ece.neu.edu.

M. V. Bunn is with the Department of Telecommunications, Federal Institute of Santa Catarina, Sao Jose, Santa Catarina, 88103-310, Brazil.

Jing Tu is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, Massachusetts 02115, USA.

for the typical plane wave excitation and free space scenario. The approach presented in this paper is more general, as it holds for arbitrary backgrounds and fields. Motivation for our research is provided by the possibility of developing novel sensors of constitutive properties of materials, as well as novel approaches for short-range optical communication systems, information storage devices, biometric readers, and scanners. Representative efforts in this area are described in [43], [44], [45], which address holographic fingerprinting sensors, in [46], [47], [48], which address hologram-based identification cards, and in [49], [50], [51] which discuss holographic scanning systems.

It is important to point out that the proposed power sensors can measure both real power (due to dissipation and scattering) as well as the complementary reactive power associated to field energy storage. Importantly, the optical theorem sensor is based on coherent signal processing so that it exploits both amplitude and phase information carried by the scattered signal. In this paper we show that the required coherent processing can be carried out analogically by means of holograms. The proposed approach is discussed in the classical holography framework ([52], ch. 9), but it can also be implemented via modern digital holography. It is important to point out that even though we focus next on the optical regime, the same results presented in the paper apply directly to quantum mechanical sensing with matter waves such as electron waves (where, on the other hand, instead of wave energy or power extinction, one deals with extinction of probability). In this connection, the presented optical theorem holography is also pertinent to change detection and tomography of fields and potentials using electron waves [53].

II. THEORETICAL BACKGROUND

We consider a general lossless nonhomogeneous background medium in which propagation and scattering can be described in the framework of the Helmholtz operator $[\nabla^2 + k^2(\mathbf{r})]$ where ∇^2 is the Laplacian and $k(\mathbf{r})$ is the space-dependent wavenumber, which is assumed to go to the free space value $k_0 = \omega/c = 2\pi/\lambda$ as $|\mathbf{r}| \equiv r \rightarrow \infty$, where ω is the angular oscillation frequency, c is the vacuum speed of light, and λ is the free space wavelength. The medium response is characterized by the Green function $G(\mathbf{r}, \mathbf{r}')$ obeying

$$[\nabla^2 + k^2(\mathbf{r})]G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (1)$$

where δ is Dirac's delta function, subject to the relevant boundary conditions. Next we focus on unbounded media formed by materials of finite extent so that the relevant boundary condition is the radiation condition. The incident field ψ_i probing the scatterer obeys

$$[\nabla^2 + k^2(\mathbf{r})]\psi_i(\mathbf{r}) = 0 \quad (2)$$

in a region Σ containing the region of interest (ROI) τ ($\tau \subseteq \Sigma$) where the scatterer or medium perturbation is located. In the presence of the scatterer or perturbation the total field ψ in Σ is given by

$$\psi(\mathbf{r}) = \psi_i(\mathbf{r}) + \psi_s(\mathbf{r}) \quad (3)$$

where ψ_s denotes the scattered field.

It can be shown from the main result eq. 15 of [23] that the average or real extincted power P_e due to both dissipation and scattering can be expressed as the projective measurement

$$P_e = -k_0^{-1} \text{Im} \int_{\mathbb{R}^3 \setminus \tau} d\mathbf{r} \psi_s(\mathbf{r}) R(\mathbf{r}) \quad (4)$$

where the exterior sensor or receiver mode R is such that when R functions as a source it generates in the ROI the complex conjugate (cc) probing field ψ_i^* , i.e.,

$$\int_{\mathbb{R}^3 \setminus \tau} d\mathbf{r}' R(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') = \psi_i^*(\mathbf{r}) \quad \mathbf{r} \in \tau. \quad (5)$$

In addition, from the reactive optical theorem in [27], [32], it also follows that the extincted reactive power P_r associated to the storage of field energy near the scatterer or perturbation is given by

$$P_r = -k_0^{-1} \text{Re} \int_{\mathbb{R}^3 \setminus \tau} d\mathbf{r} \psi_s(\mathbf{r}) R(\mathbf{r}) \quad (6)$$

where R obeys (5).

Fortunately, there exist detectors R (indeed, an infinitude of them [54]) obeying the key requirement (5). This is readily visualized with the help of the Huygens-Fresnel principle which renders a singular surface source supported in the boundary $\partial\tau$ of τ which generates the desired cc probing field ψ_i^* . This principle also provides the theoretical justification of the well-known time reversal method for the generation of the complex conjugate (time reversal) field in the background medium. Importantly, this approach benefits from the complex nature of the medium and does not require prior knowledge of it. In this approach the source of ψ_i^* consists of a time reversal mirror array whose radiating elements are located far from the ROI at positions $\mathbf{X}_n, n = 1, \dots, N$ and are driven by signals $2ik_0\psi_i^*(\mathbf{X}_n), n = 1, \dots, N$ (this follows from basic diffraction theory and is shown, e.g., in [32], [55]). When this source radiates into the medium it generates a field that is (approximately, given the diffraction limits) proportional to the cc probing field ψ_i^* in the ROI. The theoretical principles and limitations of this approach are well documented in the literature (see [32], [55], [56] and the references therein) and will not be elaborated any further here.

On the other hand, in optics, one measures only field intensities. Thus the optical implementation of a time-reversal-based optical theorem detector can be challenging. We discuss in the following a conceptually simple and highly practical holography-based approach which allows one to measure both P_e and P_r with a single optical sensor (a single-pixel camera) in combination with a lens and a hologram of the background scene. The next section outlines the basic concept for the simple case of a scatterer probed by an arbitrary field in free space, in which the detector can be based on conventional lens-based imaging. The following section of the paper expands the basic concept, by incorporating holography, making the technique applicable to more complex backgrounds or scenes that can be unknown or hidden.

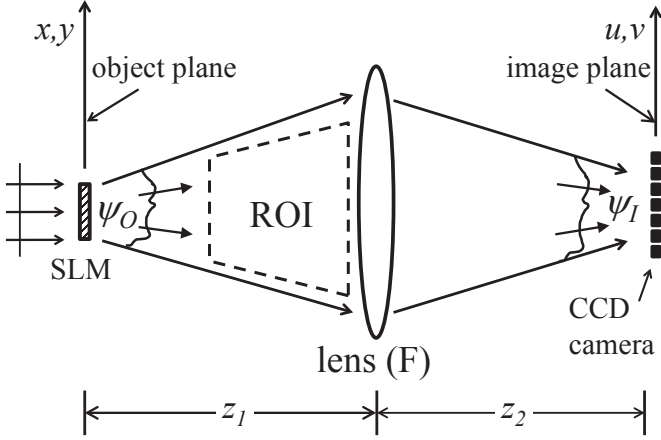


Fig. 1. Lens-based imaging system considered in the theoretical development of the lens-imaging-based optical theorem detection approach.

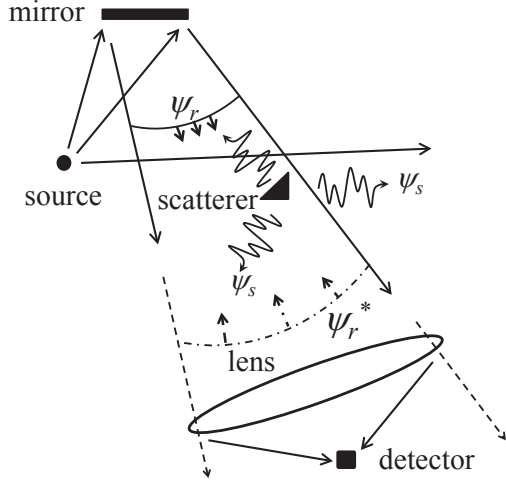


Fig. 2. Optical theorem detector corresponding to a diverging probing wave ψ_r in free space.

III. LENS-IMAGING-BASED OPTICAL THEOREM DETECTOR

As a first step to develop the new optical theorem holography, we consider the basic problem of designing an optical theorem detector that is based on conventional lens-based imaging. The developed system applies to the probing with arbitrary fields of a scatterer that is embedded in free space where the wavenumber $k_0 = 2\pi/\lambda$ where λ is the wavelength. Fig. 1 shows the geometry of the system. The field used to probe the ROI where the scatterer may be located is generated by passing an incident plane wave (laser beam) through a space-light modulator (SLM) which can operate in either transmissive (as shown in the figure) or reflective modality. We denote the field at the object plane immediately behind the SLM as ψ_O . It is assumed next that the ROI is a few wavelengths removed from the object plane so that the probing field passing by the ROI is approximately purely propagating, which allows us to describe propagation and backpropagation

or time reversal in the canonical framework of the Fresnel approximation. Using well-known results of Fourier optics ([52], secs. 5.3.2 and 5.3.3) we find that the field ψ_I at the image plane, where

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{F}, \quad (7)$$

is given by

$$\psi_I(u, v) = h(u, v) \otimes \hat{\psi}_I(u, v) \quad (8)$$

where $\otimes$ denotes convolution and h is the Fraunhofer diffraction pattern of the lens pupil, i.e.,

$$h(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' P(\lambda z_2 x', \lambda z_2 y') e^{-i2\pi(ux' + vy')}, \quad (9)$$

where P is the pupil function, and $\hat{\psi}_I$ is the ideal image field (under geometrical optics), i.e.,

$$\hat{\psi}_I(u, v) = \frac{1}{|M|} \psi_O\left(\frac{u}{M}, \frac{v}{M}\right), \quad (10)$$

where the magnification $M = -z_2/z_1$.

Furthermore, if the cc probing field $\hat{\psi}_I^*$ is measured and fed back to the imaging system (as input or “object” field at the original image plane), then the respective “image” field ψ_o at the original object plane is given by

$$\psi_o(x, y) = h'(x, y) \otimes \hat{\psi}_o(x, y) \quad (11)$$

where

$$h'(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' P(\lambda z_1 x', \lambda z_1 y') e^{i2\pi(xx' + yy')} \quad (12)$$

and

$$\hat{\psi}_o(x, y) = \psi_o^*(x, y). \quad (13)$$

It follows that if the effects of diffraction are neglected, then ψ_o is simply ψ_O^* , the cc field at the original object plane. This further implies that, at least in the ideal geometrical optics scenario, one can reconstruct the cc probing field in the ROI. In practice, the diffraction effects limit the capacity to produce the required cc probing field exactly. On the other hand, this approach renders a good approximation of the required cc field under mild conditions.

Assume now that a scatterer is present in the ROI. The scatterer is probed with the field generated by the SLM, as shown in Fig. 1. The objective is to estimate the total extincted power from intensity-only data accessible at the original image plane where a CCD camera can be placed, as shown in Fig. 1. The power budget of the corresponding scattering phenomenon can be measured using the optical theorem expressions (4,6). Note that the required receiver function R is such that when R is a source it radiates in the ROI the cc probing field. The lens analysis given above considered only the map from a field to another. It was found that when the field $\hat{\psi}_I^*$ is applied as input at the original image plane, then another field is generated at the original object plane which is approximately equal to the cc probing field ψ_O^* . Furthermore, the corresponding field in the ROI is approximately equal to the respective cc probing field there. To apply (4,6) we need to reformulate this result in

terms of an equivalent source, being of particular interest one that is supported at the original image plane and is consistent with the boundary value of the field, i.e., ψ_I^* , over that plane. It follows from basic Rayleigh-Sommerfeld diffraction theory ([57], ch. 2) that an equivalent source at the original image plane is

$$R(u, v) = 2ik_0\psi_I^*(u, v). \quad (14)$$

Substituting this in (4,6) we obtain

$$\begin{aligned} P_e &= -2\text{Re} \iint_A du dv \psi_I^*(u, v) \psi_s(u, v) \\ P_r &= -2\text{Im} \iint_A du dv \psi_I^*(u, v) \psi_s(u, v) \end{aligned} \quad (15)$$

where integration is over the available sensing aperture A (e.g., the CCD camera area).

In practice one employs a finite number of sensors (CCD detectors) or pixels as shown in Fig. 1, in which case the result (15) needs to be approximated as

$$\begin{aligned} P_e &\simeq -2K\text{Re} \sum_{n=1}^N \psi_I^*(\mathbf{X}_n) \psi_s(\mathbf{X}_n) \\ P_r &\simeq -2K\text{Im} \sum_{n=1}^N \psi_I^*(\mathbf{X}_n) \psi_s(\mathbf{X}_n) \end{aligned} \quad (16)$$

where $\mathbf{X}_n, n = 1, \dots, N$ denotes the sensor positions and K is a calibration constant representing the effective sensor area.

In the absence of the scatterer, the field intensity $I_i(\mathbf{X}_n)$ measured at the n th detector is (apart from noise) equal to

$$I_i(\mathbf{X}_n) = |\psi_I(\mathbf{X}_n)|^2. \quad (17)$$

In the presence of the scatterer, the output of the n th detector is the total intensity

$$\begin{aligned} I_t(\mathbf{X}_n) &= |\psi_I(\mathbf{X}_n) + \psi_s(\mathbf{X}_n)|^2 \\ &\simeq |\psi_I(\mathbf{X}_n)|^2 + 2\text{Re}[\psi_I^*(\mathbf{X}_n) \psi_s(\mathbf{X}_n)] \end{aligned} \quad (18)$$

where in the last statement we assume that the scatterer is weak so that $|\psi_s| \ll |\psi_I|$. If this approximation holds, then it follows from (16) that the value of the quantity

$$v_{OT} = K \sum_{n=1}^N [I_i(\mathbf{X}_n) - I_t(\mathbf{X}_n)] \simeq P_e \quad (19)$$

so that the real power P_e extincted by the scatterer upon excitation by the given probing field can be estimated directly from the intensity-only data. In contrast, the measurement of P_r via (16) is based on the same setup but requires an additional interferometer.

These general results take a simple form for the particular case of a diverging probing field $\psi_I = \psi_r$ produced by a point source or pinhole, as shown in Fig. 2. In this case, a source that generates the cc probing field ψ_r^* in the region where the scatterer is located is a point source or pinhole located along the optical axis at the focal point of the lens imaging system, as governed by the lens law. The corresponding optical theorem detector is the receiver counterpart of this transmitter system, in which the point source is substituted by a single field sensor or detector at the focal spot of the imaging system, as depicted

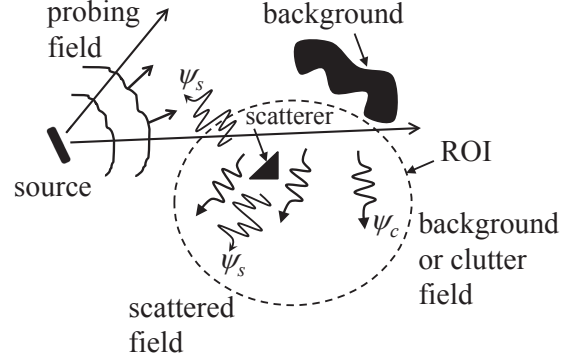


Fig. 3. Interrogation by a general probing wave of a scatterer that is embedded in a complex, possibly unknown background medium or scene.

in Fig. 2. The value of the real power P_e extincted by the scatterer is given from (19) with $N = 1$ by the intensity-only data at the single receiver. The reactive power P_r can also be measured but only with additional interferometry. The basic system in Fig. 2 will remain important in the following developments, playing the role of the essential reference-wave building block of the holographic system, and allowing us to measure the extincted power with a single detector (a single-pixel camera).

IV. OPTICAL THEOREM HOLOGRAPHY

The main goal of the rest of the paper is to develop and illustrate a holographic method to measure the total extincted power using a single sensor (a single-pixel camera). Figure 3 illustrates the general system under consideration, in which a scatterer that is embedded in a nonhomogeneous background or clutter is interrogated with an arbitrary probing field. Let ψ_p be the field produced by the source in the free space medium without the background scene. As illustrated in the figure, this probing field can be quite general. The field in the presence of the background is termed the incident field ψ_i and is given by

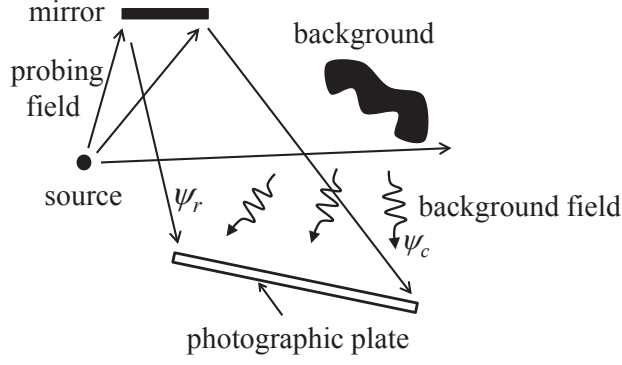
$$\psi_i = \psi_p + \psi_c \quad (20)$$

where ψ_c is field scattered by the background. Finally, in the presence of both the background and the scatterer, the total field ψ is given by

$$\psi = \psi_p + \psi_c + \psi_s = \psi_i + \psi_s \quad (21)$$

where ψ_s denotes the scattered field.

The essence of the proposed optical theorem holography is explained next for the special case of a probing beam generated by a point source, as shown in part (a) of Fig. 4(a). As shown in the figure, part of the probing beam can be relayed into a mirror for the generation of a reference wave ψ_r . As shown in part (a) of Fig. 4(a), a recording medium such as a photographic plate can be used to capture the intensity of the sum of the incident field ψ_i plus the reference wave ψ_r .



(a) Recording

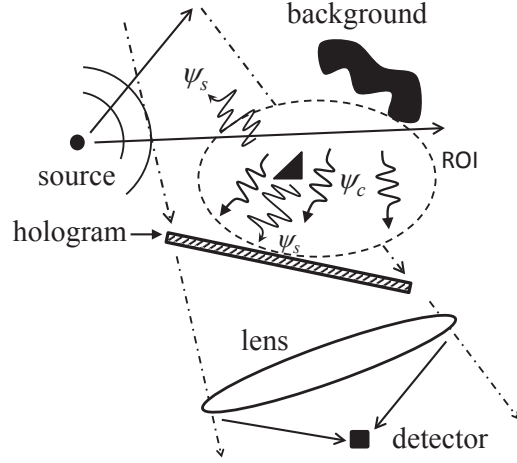
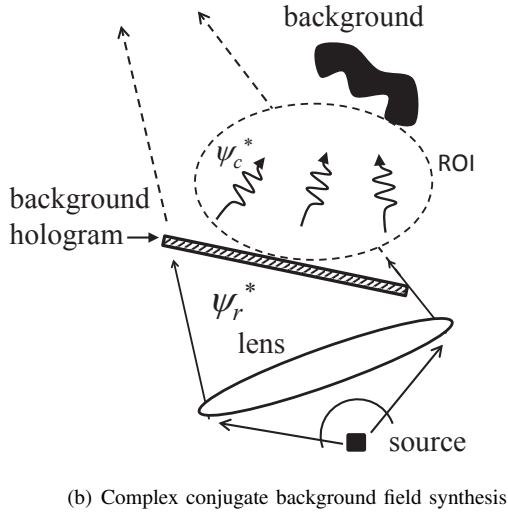


Fig. 5. Holographic optical theorem detector.



(b) Complex conjugate background field synthesis

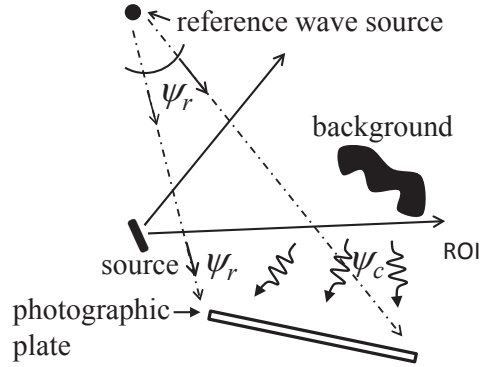


Fig. 6. Generation of the hologram using an independent reference wave source.

Fig. 4. (a) Recording of the hologram of the background scene. (b) Synthesis of the cc probing field in the ROI using a point source or pinhole at the spot where the lens-based-imaging system focuses the reference wave due to the source and reflecting mirror in part (a).

Under the reflective geometry assumed in the figure, $\psi_i = \psi_c$, so that the recording medium captures over its sensing aperture region A the intensity

$$\begin{aligned} I_{c+r}(\mathbf{r}) &= |\psi_c(\mathbf{r}) + \psi_r(\mathbf{r})|^2 \\ &= |\psi_c(\mathbf{r})|^2 + \psi_c^*(\mathbf{r})\psi_r(\mathbf{r}) + \psi_c(\mathbf{r})\psi_r^*(\mathbf{r}) + |\psi_r(\mathbf{r})|^2 \end{aligned} \quad (22)$$

of $\psi_c + \psi_r$. In particular, in the following we represent the hologram by a transparency function t over the hologram aperture region A that is equal to the intensity I_{c+r} in (22) multiplied by a measurement constant C , i.e.,

$$t(\mathbf{r}) = CI_{c+r}(\mathbf{r}) \quad \mathbf{r} \in A. \quad (23)$$

Now, if the thus recorded hologram is excited by the cc reference field ψ_r^* , then it radiates back into the medium the cc background field ψ_c^* (this is the real image field of the hologram), as shown in part (b) of the same figure. Here we note that in the absence of the photographic plate, the source can be imaged with the lens plus detector system shown in Fig. 2 in which the detector position is chosen to ensure that the lens law is obeyed. Moreover, when the detector is replaced by a source, it generates in the same free space medium the complex conjugated version ψ_r^* of ψ_r , as depicted in the figure. It follows that in the presence of the hologram, the cc background field ψ_c^* can be generated, approximately (up to the propagating field components), in the ROI, using a point source located at the focal point where the lens-based imaging

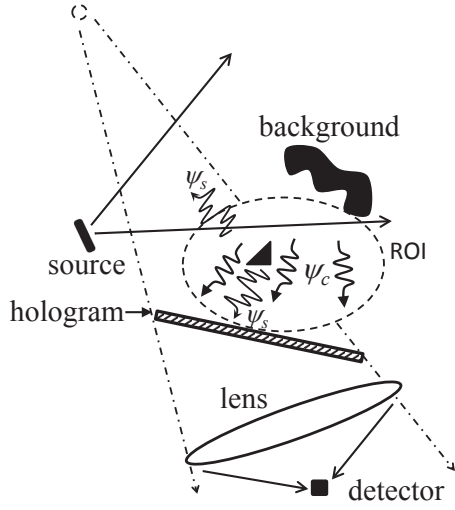


Fig. 7. Holographic optical theorem detector involving separate sources for the probing field and the reference field. The detector position is the (lens-law-defined) point where the lens focuses in free space the image of the reference wave source used to generate the hologram. Note that the latter source (shown with dashed line in the figure) is used only in the generation of the background hologram. This source is not used in the experiment to measure the extinguished power by means of the optical theorem detector.

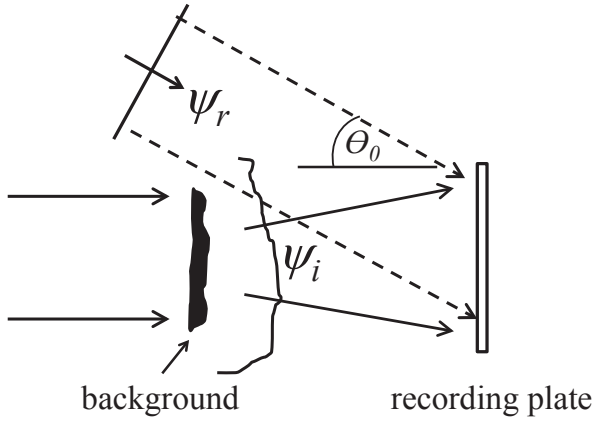


Fig. 8. Generation of the Leith-Upatnieks hologram for the background. The incidence angle of the reference plane wave is θ_0 .

system images the reference wave source which in the present case is equal to the probing field source. In practice, it suffices that the thus generated field be approximately equal to the cc background field ψ_c^* in the relevant ROI containing the scatterers or medium perturbations of interest. This procedure completes the definition of the sought-after holographic optical theorem detector for any scatterer in the ROI. The obtained holographic detector is shown in Fig. 5. Importantly, this holographic detector is nonadaptive or universal in the sense that it applies to any scatterer appearing subsequently in the ROI. To measure the extinguished power of any scatterer in the ROI, the total medium including the background plus the scatterer in the ROI is probed using the same source used to excite the background during the generation of the

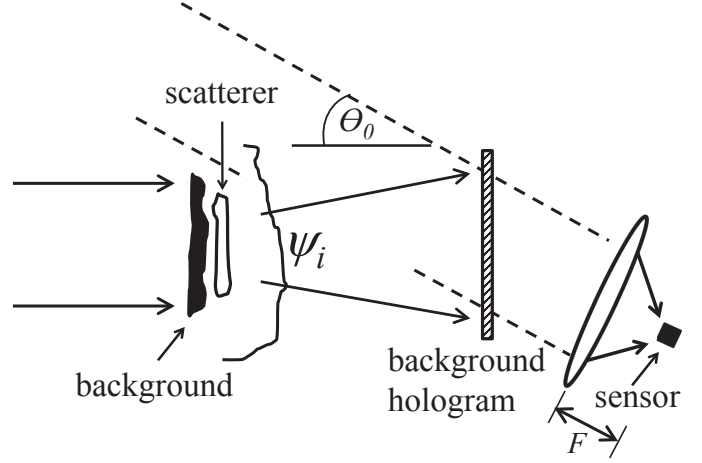


Fig. 9. Optical theorem detector based on Leith-Upatnieks-holography.

hologram. The mirror or reference wave is removed, and the total (incident plus scattered) field is sensed using the detector constituted by the hologram, the lens, and the single detector. Moreover, this approach still holds even if the reference wave cannot be removed, so long as the reference beam does not intercept the ROI containing the scatterer.

To simplify the exposition we have considered so far the special case of a point source probing the medium. This is a highly practical optical theorem experiment in which the same source can be used for probing and the generation of the reference wave. On the other hand, the same general optical theorem holography concept can be applied to arbitrary probing fields by adopting a separate source for the reference wave, as shown in Fig. 6. Of particular interest is the case of a point source or pinhole for the reference wave which ensures that the resulting optical theorem detector employs a single detector, as shown in Fig. 7. An important special case is that of plane wave excitation which corresponds to having the reference wave source at infinity. To simplify the discussion, in the remainder of this section we will focus on this particular special case. On the other hand, the general results presented next for plane reference waves remain valid for diverging spherical waves, as can be shown, e.g., via standard Fourier optics ([52], p. 119-121).

Assume that both the background scene and the ROI are several wavelengths apart from the hologram aperture region A . Then it follows from basic Rayleigh-Sommerfeld diffraction theory that an approximate version of the cc field ψ_c^* can be generated in the ROI using a source $2ik_0\psi_c^*$ supported in the hologram aperture area A . Therefore according to (4) an approximate estimate P_e' of the extinguished power P_e is given by

$$P_e \simeq P_e' = -2\text{Re} \iint_A dx dy \psi_c^*(x, y) \psi_s(x, y) \quad (24)$$

where here x, y denote the coordinates of point $\mathbf{r}$ in the area A of the hologram (see also the related result eq.(15)).

If the reference wave arriving at the photographic plate is a plane wave $\psi_r(\mathbf{r}) = A'e^{ik_0\mathbf{s}_0 \cdot \mathbf{r}}$ traveling in the direction of

the unit vector $\mathbf{s}_0$ then from (22,23) and assuming $C = 1$ the hologram transmissivity

$$t(x, y) = |\psi_c(x, y)|^2 + A' e^{ik_0 \mathbf{s}_0 \cdot \mathbf{r}} \psi_c^*(x, y) + A'^* e^{-ik_0 \mathbf{s}_0 \cdot \mathbf{r}} \psi_c(x, y) + |A'|^2 \quad \mathbf{r} \in A. \quad (25)$$

For a properly designed hologram, when the background medium is probed with the same probing field used to generate the hologram, as shown in Fig. 7, the corresponding field intensity I_c measured at the detector is given by

$$\sqrt{I_c} = \frac{|A'|}{\lambda F} \iint_A dx dy |\psi_c(x, y)|^2 \quad (26)$$

where F is the lens focal length and the integration is over the area A of the hologram. Implicit in (26) is the isolation of the key real image component (the second term) in (25) so as to effectively reproduce the cc probing field in the ROI near the background when the hologram is excited by the cc reference field ψ_r^* . This can be accomplished, e.g., via the Leith-Upatnieks approach ([52], sec. 9.4). In particular, referring to Fig. 8, if the reference wave angle $\theta_0 \geq \sin^{-1}(3B\lambda)$ where B is the bandwidth of the background, then the field produced by exciting the hologram with the cc reference field ψ_r^* is approximately equal to the cc probing field ψ_c^* near the background (apart from a multiplicative factor which can be shown from (25) and the discussion leading to (24) to be equal to $|A'|^2$, the fourth term in (25)). Under this condition, the fields due to the other three components of t in (25) (the first, third, and fourth terms), focus away from the ROI near the background (they propagate in other directions) and are essentially nonradiating with respect to the vicinity of the background. For large angles θ_0 , the virtual image corresponding to the third term can even become evanescent. Thus as a radiator, the Leith-Upatnieks hologram aperture excited by the cc field ψ_r^* effectively radiates the cc probing field ψ_c^* near the background, as desired. The receiver counterpart of this “hologram plus cc field ψ_r^* ” system is the sought-after optical theorem detector based on Leith-Upatnieks holography and is shown in Fig. 9.

Now, when there is a scatterer in the ROI the total field arriving at the hologram is the sum of the background field ψ_c plus the scattered field ψ_s , and the respective intensity I_{c+s} measured at the detector is given by

$$I_{c+s} = |\sqrt{I_c} + V|^2 \quad (27)$$

where

$$V = \frac{|A'|}{\lambda F} \iint_A dx dy \psi_c^*(x, y) \psi_s(x, y) \quad (28)$$

so that if the scattered field is weak relative to the background field then from (26,27,28)

$$I_{c+s} - I_c \simeq 2 \left(\frac{|A'|}{\lambda F} \right)^2 \text{Re} \iint_A dx dy \psi_c^*(x, y) \psi_s(x, y). \quad (29)$$

Finally, comparing (29) with (24) we conclude that an estimate P_e'' of P_e' , and consequently of P_e , can be measured from the intensity-only data captured by the single sensor of the optical

theorem detector (which is located at the focal spot of the reference wave imaging system, see Fig. 9) via the formula

$$P_e \simeq P_e' \simeq P_e'' = - \left(\frac{\lambda F}{|A'|} \right)^2 (I_{c+s} - I_c). \quad (30)$$

V. ILLUSTRATION

We illustrate the previous developments for the system in two-dimensional space shown in Fig. 10. This canonical example models a basic reflective geometry relevant for certain applications such as optical scanners, biometric readers, and synthetic aperture radar systems, in which one is interested in detecting irregularities or changes over a surface (see [43], [44], [45], [46], [47], [48], [49], [50], [51]). In the geometry considered, a point source is used to probe the background which is modeled as a perfectly reflecting flat surface or mirror. As shown in the figure, the background field ψ_c consists of the superposition of the original probing field plus the reflected field which can be readily computed via the method of images. In addition, the reference field ψ_r arriving at the hologram aperture is provided by the same point source used to probe the scene. In the following numerical results we considered a hologram aperture of length $L = 400\lambda$ centered at the origin of coordinates ($x = 0, y = 0$). The mirror was located at $y = -1500\lambda$. The probing source was located at ($x = -400\lambda, y = -500\lambda$). The ROI was the rectangular region $\tau = \{x \in \mathbb{R}, y \in \mathbb{R}, |x + 240\lambda| \leq R, y - 1500\lambda \geq R\}$ where R characterizes the size of the ROI. In the simulations we used $R = 80\lambda$. In the scattering computations we considered a scatterer for which an exact analytical solution of the scattering problem is available. In particular, we used a perfectly reflecting semicircle of radius $a < R$ centered at ($x = -240\lambda, y = -1500\lambda$). This scatterer is illustrated in Fig. 10, and the corresponding exact scattering computations for a unit-amplitude point source are provided in Appendix A.

Figure 11 shows plots of the real part of the reflective component of the background field in the boundary $y = -1500\lambda$ of the ROI (solid line), and of the real part of the incident component of the field generated in the same boundary by the hologram aperture upon excitation by the cc reference wave (dotted line). The two plots are very similar, indicating that the hologram successfully generates the cc background field in the ROI, as desired. In the latter plot, the value of the field due to the hologram is multiplied by a corrective factor corresponding to $1/|A'|^2$ (see the discussion following (26)), which is estimated numerically as the reciprocal of the average value of the intensity of the reference field over the hologram aperture. In the presence of the background mirror, the total field due to the hologram aperture plus cc reference wave source is given by the sum of this incident component plus a companion reflective part generated at the mirror. Figure 12 provides plots of the imaginary part of the complex conjugate version of the reflective component of the background field in the boundary $y = -1500\lambda$ of the ROI (solid line), and of the imaginary part of the incident component of the field generated in the same boundary by the hologram aperture upon excitation by the cc reference wave (dotted line). In the latter plot, the field produced by the hologram is multiplied by the

same corrective factor ($1/|A'|^2$) involved in the companion (real part) plot in Fig. 11. Figures 11 and 12 convincingly show that the hologram, when acting as a source, effectively generates the cc background field in the ROI, as is required in order for this holographic sensor to perform the optical theorem operation of measuring the extincted power.

Central to the successful holographic implementation of the optical theorem is the effective isolation in the ROI of the field produced by the real image component of the hologram (the second term in (25)). Figure 13 shows that this key requirement is achieved in this example. In particular, the figure provides plots of the amplitude of the incident fields generated in the boundary $y = -1500\lambda$ of the ROI corresponding to three components of the hologram aperture upon excitation by the cc reference field: The solid line plot corresponds to the amplitude of the real image field (second term in (25)). The dashed line plot corresponds to the virtual image field (third term in (25)). The dotted line corresponds to the DC or constant term associated to the intensity of the reference field alone (fourth term in (25)). The amplitude of the key real image field is significantly larger than that of the two other components, as desired. The field amplitude of the first term in (25) is not shown, but its value is comparable in this example to that of the virtual image. These results illustrate the successful isolation, in the ROI, of the key real image field of the hologram. This is accomplished in this example by putting the reference beam source away from the main optical axis, in analogy with the classical Leith-Upatnieks holography.

The solid line in Fig. 14 illustrates the variation of the scattered power P_s versus the scatterer radius a in the $0 - 4\lambda$ range. The scattered power P_s was evaluated using (38). The same figure shows the plot corresponding to the extincted power as measured by the holographic optical theorem detector. Both plots are almost identical, as expected from Figs. 11,12,13 and the optical theorem holography theory presented in the paper. Figure 15 shows the percentage of error of the holographic optical theorem detector versus the scatterer radius a in the larger $0 - 160\lambda$ range. The error is defined as $\text{error}(\%) = 100|P'_s - P''_e|/P'_s$, where P''_e is defined in (30) while P'_s is defined in (38). For radius smaller than 80λ , corresponding to a scatterer confined within the ROI τ , the error percentage remains below 5%. This is consistent with the plots in Figs. 11 and 12, which show that the cc background field is well approximated in this region by the transmit form of the optical theorem hologram. Thus this hologram obeys the basic requirement that allows it to function as extinction power sensor for scatterers residing in this ROI. On the other hand, as the scatterer radius increases beyond the critical radius of 80λ the sensing error grows quickly. This is expected from the loss of imaging capability, due to diffraction, of the finite-sized hologram aperture in the correspondingly larger scattering region, which implies that the hologram can no longer function (for $a > 80\lambda$) as extinction power sensor.

VI. CONCLUSION

The optical theorem describes the energy budget of wave scattering phenomena. It is a basic principle that can be used

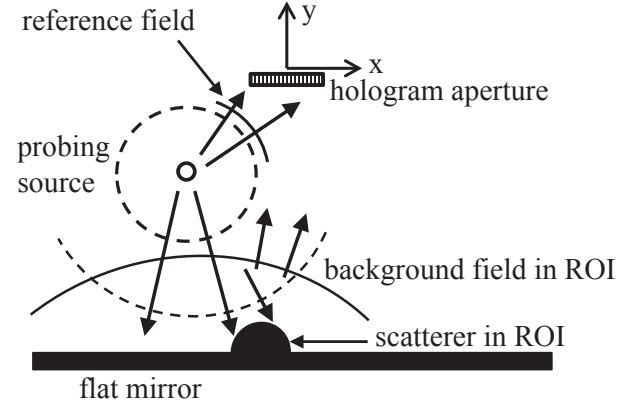


Fig. 10. System considered in the example.

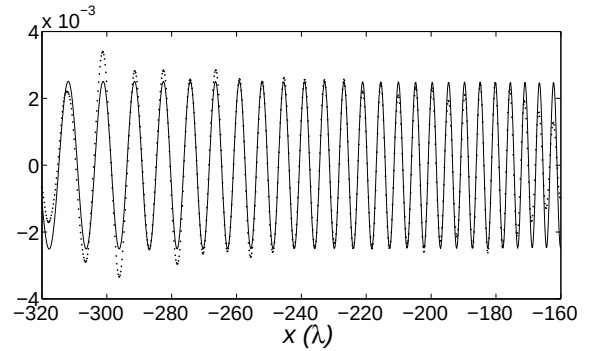


Fig. 11. Plot of the real part of the reflective component of the background field in the boundary $y = -1500\lambda$ of the ROI (solid line), and of the real part of the incident component of the field generated in the same boundary by the hologram aperture upon excitation by the cc reference wave (dotted line).

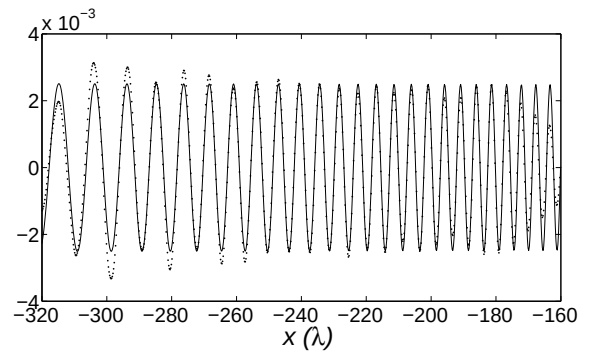


Fig. 12. Plot of the imaginary part of the complex conjugate version of the reflective component of the background field in the boundary $y = -1500\lambda$ of the ROI (solid line), and of the imaginary part of the incident component of the field generated in the same boundary by the hologram aperture upon excitation by the cc reference wave (dotted line).

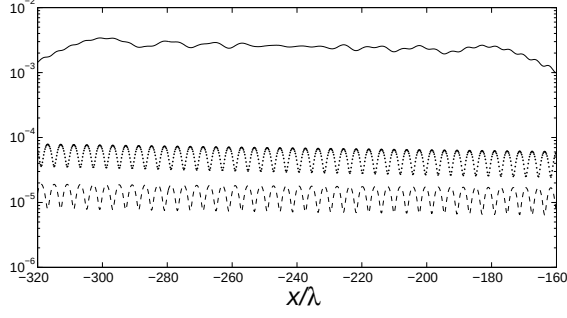


Fig. 13. Base-10 semilog plots of the amplitude of the incident fields generated in the boundary $y = -1500\lambda$ of the ROI by three components of the hologram aperture upon excitation by the cc reference field: Amplitude of the real image (solid line), of the virtual image (dashed line), and of the DC or constant term associated to the intensity of the reference field alone (dotted line).

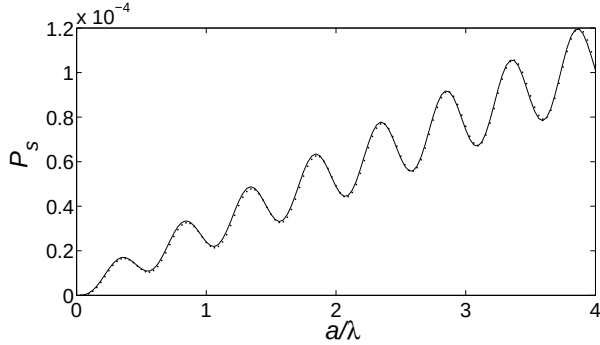


Fig. 14. Dotted line: Plot of the scattered power P_s based on the analytical expression (38) versus the scatterer radius a . Solid line: Plot of the extincted power measured by the holographic optical theorem detector versus a .

to design sensors of the power extincted by a scatterer that is immersed in a given scene or background upon excitation by a given probing field. In this work we have shown how to design optical theorem power sensors that are based on classical holography. The derived optical theorem holography broadens the perspectives on the practical implementation of the optical theorem. It further facilitates the development of optical theorem sensors using optical fields and quantum-mechanical matter waves in which one measures directly only field intensities.

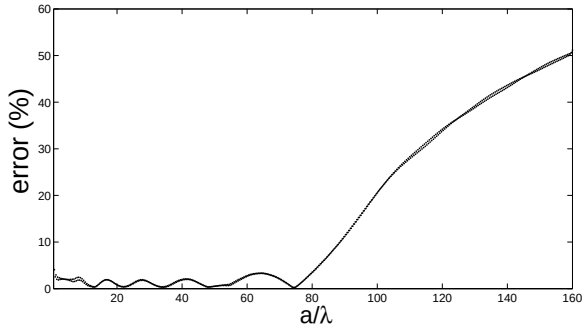


Fig. 15. Percentage of error of the holographic optical theorem detector versus the scatterer radius a .

Possible applications include novel holography-based power sensors, detectors, scanners, communication systems, and information storage devices based on photonics and quantum electronics.

As part of the theoretical development, we detailed the lens-imaging-based implementation of the optical theorem in free space. This included the general case using arbitrary propagating fields as well as a special case using point source excitation in which the optical theorem sensor can be implemented with a single-detector (a single-pixel camera). These results on the free-space lens-imaging-based implementation were applied later to the development of the more general optical theorem holography which, unlike the lens-imaging-based method, applies, in principle, to arbitrary scenes or backgrounds. In the derived optical theorem holography, information about the background (without the scatterer) is captured in the optical theorem hologram which is created by the interference of the background field with a known reference wave. The optical theorem hologram acts as the entrance coded aperture of a lens-based detector that is placed so as to image the reference wave source. The resulting hologram plus lens-based detector system is the sought-after holography-based optical theorem sensor. It is a nonadaptive sensor that measures the power extincted by any scatterer appearing in the neighborhood of the background. It performs this function well, so long as its transmit counterpart effectively images the cc probing field in the scatterer region. This was illustrated in the paper with an example in two-dimensional space.

APPENDIX A

Let (ρ', ϕ') and (ρ, ϕ) be the cylindrical coordinates of the point source and the observation point, respectively. Here the origin is the center of the semicircular scatterer of radius a . It is not hard to show using standard Green's function techniques that the relevant background Green function G_b for the background medium bounded by the mirror, represented mathematically by homogeneous Dirichlet boundary condition for the field at the plane $y = 0$, is

$$G_b(\rho, \phi; \rho', \phi') = i \sum_{m=1}^{\infty} H_m^{(1)}(k_0 \rho_{>}) J_m(k_0 \rho_{<}) \sin(m\phi) \sin(m\phi') \quad (31)$$

where J_m is the Bessel function of order m , $H_m^{(1)}$ is the Hankel function of the first kind of order m , $\rho_{>} = \max(\rho, \rho')$, and $\rho_{<} = \min(\rho, \rho')$. This Green function defines the probing field in the region under investigation. On the other hand, total Green's function G_t which corresponds to the total (incident plus scattered) field in the same region can be shown to be given by

$$G_t(\rho, \phi; \rho', \phi') = i \sum_{m=1}^{\infty} \left[J_m(k_0 \rho_{<}) - \frac{J_m(k_0 a)}{H_m^{(1)}(k_0 a)} H_m^{(1)}(k_0 \rho_{<}) \right] H_m^{(1)}(k_0 \rho_{>}) \sin(m\phi) \sin(m\phi'). \quad (32)$$

Thus from (31,32) the scattering Green function G_s corresponding to the scattered field is given by

$$G_s(\rho, \phi; \rho', \phi') = -i \sum_{m=1}^{\infty} H_m^{(1)}(k_0 \rho') H_m^{(1)}(k_0 \rho) \frac{J_m(k_0 a)}{H_m^{(1)}(k_0 a)} \sin(m\phi) \sin(m\phi'). \quad (33)$$

It follows from (33) and the asymptotic behavior

$$H_m^{(1)}(k_0 \rho) \sim \sqrt{\frac{2}{\pi k_0 \rho}} e^{ik_0 \rho} e^{-im\pi/2} e^{-i\pi/4} \quad k_0 \rho \rightarrow \infty \quad (34)$$

that for far zone observation points

$$G_s(\rho, \phi; \rho', \phi') \sim -i C_0 \sum_{m=1}^{\infty} (-i)^m H_m^{(1)}(k_0 \rho') \frac{J_m(k_0 a)}{H_m^{(1)}(k_0 a)} \sin(m\phi) \sin(m\phi') \quad (35)$$

where

$$C_0 = \sqrt{\frac{2}{\pi k_0 \rho}} e^{ik_0 \rho} e^{-i\pi/4}. \quad (36)$$

Using (35) we can evaluate the far-zone scattered power

$$\begin{aligned} P_s &= \int_0^\pi \rho d\phi |G_s(\rho, \phi; \rho', \phi')|^2 \\ &= k_0^{-1} \sum_{m=1}^{\infty} |H_m^{(1)}(k_0 \rho')|^2 \frac{[J_m(k_0 a)]^2}{|H_m^{(1)}(k_0 a)|^2} \sin^2(m\phi'). \end{aligned} \quad (37)$$

Finally, the Bessel functions $J_m(x)$ decay exponentially fast for $m > x$. Thus the series expansion in (37) can be effectively truncated, rendering the following approximate value P'_s of P_s :

$$P_s \simeq P'_s = k_0^{-1} \sum_{m=1}^M |H_m^{(1)}(k_0 \rho')|^2 \frac{[J_m(k_0 a)]^2}{|H_m^{(1)}(k_0 a)|^2} \sin^2(m\phi'), \quad (38)$$

where $M = [k_0 a] + p$ where $p > 0$ is a small integer. We used $p = 5$ in the computer simulations.

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Edwin A. Marengo received the Bachelor's degree in Electromechanical Engineering (Valedictorian and Summa Cum Laude) from the Technological University of Panama, Panama City, in 1990, and the M.S. and Ph.D. degrees in Electrical Engineering from Northeastern University, Boston, MA, in 1994 and 1997, respectively. His Ph.D. dissertation, titled A New Theory for Analysis and Synthesis of Time-dependent Scalar and Electromagnetic Sources and Fields, was supervised by Professor Anthony Devaney. From 1997 to 2004 he did post-doctoral work at Northeastern University, the University of Arizona, the Technological University of Panama, and Arizona State University, investigating a broad spectrum of topics in electrical engineering, physics, and mathematics (including computational methods in electromagnetics, quantum information theory, wireless communications, and time-reversal imaging). Since 2004 he has been with the Department of Electrical and Computer Engineering at Northeastern University where he is an Associate Professor. His research focuses on theoretical and computational aspects of electromagnetic imaging, scattering, inverse scattering, and change detection, as well as physics-based signal processing and information theory. Dr. Marengo is an associate editor of the *IEEE Transactions on Computational Imaging* and a board member of the SpringerPlus Journal Editorial Board. In the past, he served as Associate Editor of the *IEEE Transactions on Image Processing* and the *Scientific World Journal*. He is a member of the International Union of Radio Science (URSI) (elected into Commission B: Fields and Waves, in 2006), a senior member of the IEEE and a member of the Optical Society of America, as well as a member of the Honor Societies of Phi Kappa Phi, Eta Kappa Nu, and Tau Beta Pi. He has been a Fulbright scholar sponsored by the U.S. Department of State, and is a recipient of the National Science Foundation (NSF) CAREER award.



Marcus V. Bunn is a student of Telecommunications Engineering at the Federal Institute of Technology, Santa Catarina, Brazil. Thanks to a CAPES Foundation scholarship, he spent academic year 2014-2015 at the University of California at Santa Barbara and Northeastern University, where he took courses in signal processing and did research in optical imaging. His research interests include digital signal processing and Fourier optics.



Jing Tu obtained the B.S. degree in Optoelectronics Engineering from Huazhong University of Science and Technology, China in 2011, and the M.S. degree in Electrical Engineering from Northeastern University, Boston, MA, in 2013. He is currently a Ph.D. candidate in Electrical Engineering at Northeastern University and will graduate in May 2016. His research interests include the areas of electromagnetic theory, scattering theory, physics-based detection theory, optics, imaging, and computational methods in electromagnetics.